

# 2.1

## Comparing Fractions

### YOU WILL NEED

- coloured pencils
- a number line

### GOAL

Compare and order fractions using benchmarks and equivalent fractions.

### LEARN ABOUT *the Math*

Sarah's math teacher has a new way to decide which students will present their projects first. In groups of 10, each student pulls a slip of paper from a jar. A fraction or **mixed number** is written on each slip. The student with the fourth greatest number will present first.

In Sarah's group, the slips were

$2\frac{1}{3}$	$\frac{8}{9}$	$\frac{12}{5}$	$\frac{2}{9}$	$\frac{15}{18}$	$\frac{7}{9}$	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{2}{5}$	$\frac{4}{5}$
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Sarah chose  $\frac{8}{9}$ .

The students decided to place the fractions on a number line to help them see the order.



### lowest terms

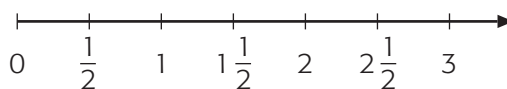
an equivalent form of a fraction with a numerator and a denominator that have no common factors other than 1; for example,  $\frac{3}{4}$  is the lowest term form of  $\frac{12}{16}$ , since  $\frac{3}{4} = \frac{12}{16}$ , and 3 and 4 have no common factors other than 1

### common denominator

a common multiple of two or more denominators; for example, a common denominator for  $\frac{2}{3}$  and  $\frac{3}{6}$  would be any multiple of 6. If you use the least common multiple of the denominators, the common denominator is called the least common denominator

## Will Sarah present first in her group?

- A. How do you know that  $\frac{2}{5}$  is to the left of  $\frac{4}{5}$  on the number line?
- B. How would you decide where  $\frac{2}{9}$  goes?
- C. How would writing  $\frac{4}{6}$  in **lowest terms** help you place it on the number line?
- D. How would renaming  $\frac{12}{5}$  as a mixed number help you place it on the number line?
- E. How would writing  $\frac{4}{6}$  and  $\frac{15}{18}$  as equivalent fractions with a **common denominator** help you place  $\frac{15}{18}$  on the number line?
- F. Place all the fractions from Sarah's group on the number line.



- G. Which fraction is fourth greatest? Will Sarah present first in her group?

### Reflecting

- H. Before you placed the numbers on the number line, how might you have known that the student holding  $\frac{2}{9}$  or  $\frac{2}{5}$  had no chance of presenting first?
- I. What strategies did you use to place the numbers on the number line?



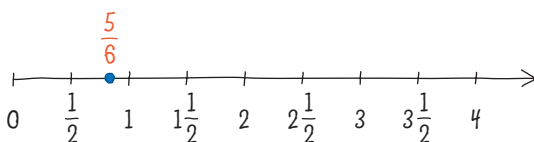
## WORK WITH the Math



### Example 1 Ordering numbers on a number line

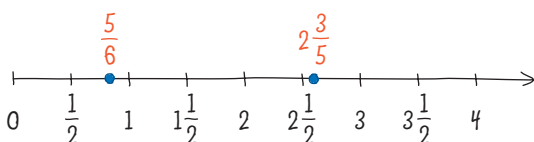
Place these numbers on a number line:  $\frac{5}{6}$ ,  $2\frac{3}{5}$ ,  $\frac{31}{8}$ ,  $\frac{2}{8}$ ,  $\frac{3}{9}$ .

#### Sarah's Solution

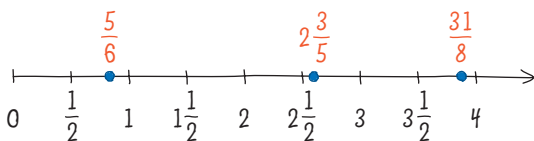


I went through the numbers from left to right.

I know that  $\frac{5}{6}$  is more than  $\frac{1}{2}$  but less than 1.



I know that  $2\frac{3}{5}$  is a bit more than  $2\frac{1}{2}$ , since  $\frac{3}{6} = \frac{1}{2}$  and  $\frac{3}{5} > \frac{3}{6}$ . I know this since each fifth is more than each sixth.

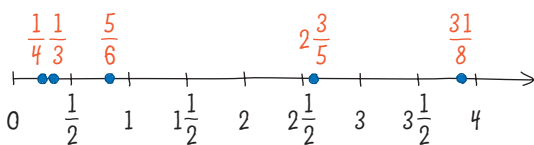


I know that  $\frac{32}{8} = 4$ , so  $\frac{31}{8}$  is a little less than 4.

$$\begin{array}{c} \div 2 \\ \frac{2}{8} = \frac{1}{4} \\ \div 2 \end{array} \quad \begin{array}{c} \div 3 \\ \frac{3}{9} = \frac{1}{3} \\ \div 3 \end{array}$$

I divided  $\frac{2}{8}$  and  $\frac{3}{9}$  by common factors to rename them in lower terms.

I know that 1 fourth piece is less than 1 third piece, so  $\frac{1}{4} < \frac{1}{3}$ .



I know that both  $\frac{1}{4}$  and  $\frac{1}{3}$  are less than  $\frac{1}{2}$ , since 1 piece out of 4 or 1 piece out of 3 is less than 1 piece out of 2.



## Example 2 | Locating a fraction between fractions

Name some fractions that are between  $\frac{1}{2}$  and  $\frac{2}{3}$ .

### Ryan's Solution

$$\begin{array}{l} \begin{array}{c} \times 3 \\ \frac{1}{2} = \frac{3}{6} \\ \times 3 \end{array} \quad \begin{array}{c} \times 2 \\ \frac{2}{3} = \frac{4}{6} \\ \times 2 \end{array} \end{array}$$

I renamed  $\frac{1}{2}$  and  $\frac{2}{3}$  using a common denominator. I couldn't think of a fraction between  $\frac{3}{6}$  and  $\frac{4}{6}$ , so I used equivalent fractions with a common denominator of 24 so that the numerators were farther apart.

$$\begin{array}{l} \begin{array}{c} \times 12 \\ \frac{1}{2} = \frac{12}{24} \\ \times 12 \end{array} \quad \begin{array}{c} \times 8 \\ \frac{2}{3} = \frac{16}{24} \\ \times 8 \end{array} \end{array}$$

Some fractions between  $\frac{1}{2}$  and  $\frac{2}{3}$  are  $\frac{13}{24}$ ,  $\frac{14}{24}$  or  $\frac{7}{12}$ , and  $\frac{15}{24}$  or  $\frac{5}{8}$ .

### A Checking

1. Write each pair of fractions as equivalent fractions with a common denominator.

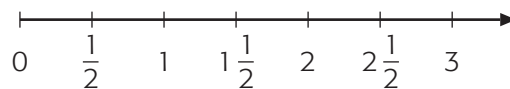
a)  $\frac{3}{5}$  and  $\frac{2}{4}$

c)  $\frac{2}{10}$  and  $\frac{1}{15}$

b)  $\frac{5}{8}$  and  $\frac{3}{4}$

d)  $\frac{2}{3}$  and  $\frac{1}{8}$

2. a) Place  $1\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{3}{5}$ , and  $\frac{7}{5}$  on the number line.



b) List the fractions in order from least to greatest.

3. Compare each pair of fractions using a strategy of your choice.

a)  $\frac{3}{7}$  and  $\frac{2}{3}$

b)  $\frac{2}{5}$  and  $\frac{1}{2}$

c)  $\frac{8}{6}$  and  $\frac{4}{8}$

### B Practising

4. Rewrite each fraction in lowest terms.

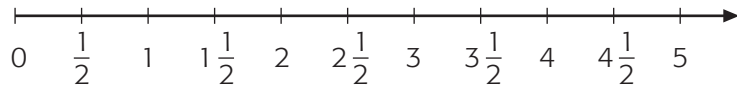
a)  $\frac{4}{8}$

b)  $\frac{10}{15}$

c)  $\frac{15}{6}$

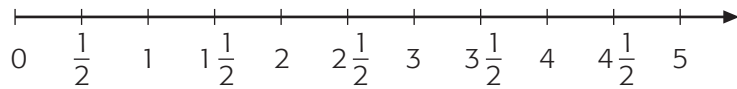
d)  $\frac{10}{6}$

5. a) Place  $\frac{15}{4}$ ,  $2\frac{2}{5}$ ,  $\frac{34}{10}$ ,  $\frac{5}{8}$ , and  $\frac{6}{9}$  on the number line.



- b) List the fractions in order from least to greatest.

6. a) Place  $2\frac{2}{5}$ ,  $3\frac{1}{2}$ ,  $\frac{8}{7}$ ,  $\frac{7}{8}$ , and  $\frac{4}{5}$  on the number line.



- b) List the fractions in order from greatest to least.

7. Compare each pair of fractions using different strategies.

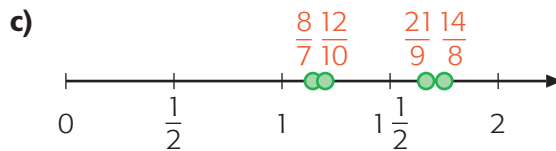
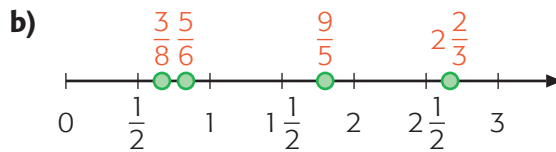
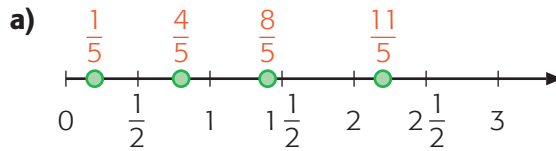
a)  $\frac{4}{9}$  and  $\frac{5}{6}$       b)  $\frac{4}{5}$  and  $\frac{1}{6}$       c)  $\frac{8}{3}$  and  $\frac{13}{15}$

8. Which number in each list is out of order?

a)  $\frac{1}{6}$ ,  $\frac{2}{5}$ ,  $\frac{4}{9}$ ,  $\frac{3}{8}$ ,  $\frac{9}{5}$       c)  $\frac{1}{10}$ ,  $\frac{4}{7}$ ,  $\frac{7}{6}$ ,  $\frac{2}{3}$ ,  $\frac{8}{5}$

b)  $\frac{12}{5}$ ,  $\frac{11}{3}$ ,  $2\frac{1}{2}$ ,  $\frac{11}{4}$ ,  $\frac{11}{2}$       d)  $\frac{3}{4}$ ,  $\frac{2}{10}$ ,  $\frac{11}{12}$ ,  $\frac{6}{5}$ ,  $\frac{3}{2}$

9. Which fraction is in the wrong location?



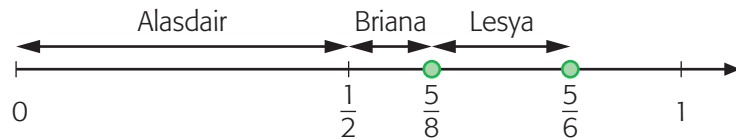
10. On which quiz did Jane do best?

Quiz	A	B	C
Score	$\frac{31}{40}$	$\frac{25}{30}$	$\frac{20}{25}$

### Reading Strategy

Look for the important information in this problem.  
Record your ideas.

11. Mike's test marks kept going up by 2, but so did the total possible score on the tests. Were his marks improving? Explain.
12. Alasdair, Briana, and Lesya played a series of chess games. They reported what fraction of the games they won:
- Alasdair said that he won less than  $\frac{1}{2}$  of his games.
  - Briana said that she won between  $\frac{1}{2}$  and  $\frac{5}{8}$  of her games.
  - Lesya said that she won between  $\frac{5}{8}$  and  $\frac{5}{6}$  of her games.



Name two possible fractions for each student.

13. Choose two fractions in which the numerators and denominators are both more than 2 apart; for example,  $\frac{3}{5}$  and  $\frac{7}{10}$ .
- Create a new fraction by using a numerator between the two numerators and a denominator between the two denominators; for example,  $\frac{5}{8}$ .
  - How does the new fraction compare with the original two fractions?
  - Try some more examples. Does this result always seem to be true?
14. How can you tell whether a fraction is greater than  $\frac{1}{2}$ ?
15. Why is it easier to compare  $\frac{2}{3}$  with  $\frac{2}{7}$  than it is to compare  $\frac{2}{3}$  with  $\frac{4}{7}$  using mental strategies?